C 81337 (Pages : 3) Name.......................................

Reg. No...................................

FOURTH SEMESTER P.G. DEGREE EXAMINATION, MARCH 2020

(CCSS)

Mathematics

MAT 4E 04—ALGEBRAIC GRAPH THEORY

(2017 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 *marks.*

1. If x is a vertex of the graph X and g is an automorphism of X, then prove that the vertex $y = x^g$ has the same valency as x, where $y = x^g$ denote the image of the element x under g.
2. Define self dual graphs and give an example of it.
3. Prove that stabilizer of an element in a permutation group V is a subgroup of V.
4. Define asymmetric graph and give an example of it.
5. Is every edge transitive graph vertex transitive ? Justify your answer.
6. Prove that a graph that contains a perfect matching has even number of vertices.

($6 \times 2 = 12$ marks)

Part B

Answer any **five** *questions.*
Each question carries 4 *marks.*

7. Find the automorphism group of the complete graph K_3.
8. Show that any edge in a bipartite graph X is a retract of X.
9. If X is a regular graph with valency k, then prove that the line graph L (X) is regular with valency $2k - 2$.

Turn over

10. Let X be a graph on n vertices. Prove that the size of the automorphism class containing X is

$$\frac{n!}{|\mathrm{Aut}(X)|}$$

11. Is Sym (3) primitive ? Justify your answer.

12. Prove that the Cayley graph $X(G, C)$ is vertex transitive.

13. Let X be a vertex and edge transitive graph. If X is of odd valency, then prove that X is arc transitive.

14. Let A be a fragment of a graph X. Prove that $N(A) = N(\bar{A})$ and $\bar{\bar{A}} = A$.

(5 × 4 = 20 marks)

Part C

Answers A **or** *B of the following questions.*
Each question carries 16 *marks.*

Unit I

15. A (a) Prove that the chromatic number of a graph X is the least integer r such that there is a homomorphism from X to K_r, where K_r denote the complete graph on r vertices.

(b) Let X be a graph which has a clique of size k. Prove that any k-coloring of X determines a retraction onto the clique.

B (a) If $v \geq k \geq i$, then prove that $\mathrm{Aut}(J(v, k, i))$ contains a subgroup isomorphic to Sym (v), where $J(v, k, i)$ denote a Johnson graph.

(b) If the line graph of a connected graph X is regular, then prove that X is regular or bi-partite and semiregular.

Unit II

16. A Prove that almost all graphs are asymmetric.

B (a) Let G be a permutation group on the set V. Prove that the number of orbits of G on V is equal to the average number of points fixed by an element of G.

(b) Let G be a transitive permutation group on V. Prove that G is primitive if and only if each nondiagonal orbit is connected.

UNIT III

17. A (a) Let X be an edge-transitive graph with no isolated vertices. If X is not vertex transitive, then prove that X is a bipartite graph.

(b) If S is an edge atom of a graph X, then prove that $2|S| \leq |V(X)|$.

B (a) Prove that the k-cube Q_k is vertex transitive.

(b) Let X is a connected vertex-transitive graph, then prove that its edge connectivity is equal to its valency.

(3 × 16 = 48 marks)