C 81338 (Pages : 3) Name.......................................

Reg. No...................................

FOURTH SEMESTER P.G. DEGREE EXAMINATION, MARCH 2020

(CCSS)

Mathematics

MAT 4E 05—ALGEBRAIC TOPOLOGY

(2017 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 *marks.*

1. Verify whether (1, 0), (1, 1) and (1, 2) are geometrically independent in $\mathbb{R}^2$.
2. Let K be the closure of a 2—simplex $\sigma = < a_0, a_1, a_2 >$. List all the elements of K.
3. Find the Betty number $R_0(K)$ where K is the closure of the 2—simplex $< a_0, a_1, a_2 >$.
4. Let K be the closure of a 2—simplex $< a_0, a_1, a_2 >$ and L be the complex

   $\{< a_0, a_1 >, < a_0, a_2 >, < a_0 >, < a_1 >, < a_2 >\}$. Give a simplicial map from K to L.
5. Let $\alpha(t) = t$ and $\beta(t) = t^2$ be paths in $\mathbb{R}$. Give a homotopy between α and β.
6. Give a path in $\mathbb{R}^2$ from (0, 0) to (1, 2).

(6 × 2 = 12 marks)

Part B

Answer any **five** *questions.*
Each question carries 4 *marks.*

7. List all faces of the 2—simplex $\sigma = < a_0, a_1, a_2 >$.
8. Describe a triangulation of the circle in $\mathbb{R}^2$.
9. Describe the incidence number $\left[\sigma^{p+1}, \sigma^p\right]$ where σ^{p+1} is a $p+1$ simplex and σ^p is a p—simplex.
10. Find the Euler characteristic $\chi\left(S^2\right)$ of the 2—sphere S^2.
11. Find the homology groups $H_0\left(S^n\right)$ for $n = 1$ and $n = 2$.

12. Let K be the closure of a 2—simplex $< a_0, a_1, a_2 >$. Describe $ost(a_0)$ and verify whether $\frac{1}{2}(a_0 + a_1)$ belongs to $ost(a_0)$.

13. Let α be the constant path at x_0 and β be a path with initial point x_0 - Show that $\alpha * \beta$ is homotopic to β.

14. Verify whether $\mathbb{R}$ is simply connected.

(5 × 4 = 20 marks)

Part C

Answer **either** *part A* **or** *part B of each of the* **three** *questions.*
Each question carries 16 *marks.*

15. A (a) Define triangulation of a topological space.

(b) Describe a triangulation of the Mobius strip.

(c) Give an example of a subspace of $\mathbb{R}^2$ which is not triangulable.

B (a) Describe the homology groups of an oriented complex K.

(b) Let K be the closure of a 2—simplex $\langle a_0, a_1, a_2 \rangle$ with $a_0 < a_1 < a_2$. Describe

i) the chain group $C_0(K)$.

ii) the boundary group $B_0(K)$.

iii) the cycle group $Z_0(K)$.

16. A (a) Define Betty number of a simplicial complex.

(b) Let K be an oriented geometric complex of dimension n. For $p = 0, 1, \ldots n$, let α_p denote the number of p—simplexes of K. Show that

$$\sum_{p=0}^{n} (-1)^p \alpha_p = \sum_{p=0}^{n} (-1)^p R_p(K)$$

where $R_p(K)$ is the p^{th} Betti number of K.

B (a) Define chain map between two chain complexes.

(b) Let $(\phi_p) : p \geq 0$ be a chain map between chain complexes of K and L. Show that

i) ϕ_p maps $B_p(K)$ into $B_p(L)$ for all p.

ii) ϕ_p maps $Z_p(K)$ into $Z_p(L)$ for all p.

17. A (a) Define homotopy of paths.

(b) Show that homotopy of paths is an equivalence relation on the set of all paths in a topological space X.

B (a) Define simply connected space.

(b) Give an example of a simply connected space.

(c) Show that every contractible space is simply connected.

(3 × 16 = 48 marks)